# A' LEVEL – INTERGRATION

©JK BEGUMISA

jkbegumisa2025@gmail.com

+256786108453/+256700798389

- *This Manual contains Revision Questions about A' level intergration.*
- *It's to boost a learner to practice more in regard to the topic, become more confident and realize that He/She can be a Master Guide in Integration.*
- *Mind you, group discussions and inter-students' revision approaches work far better than just individual Efforts.*
- ***<u>Jk Begumisa</u>*** *is responsible for any Error in this Master piece thus He promptly drops His apology.*

A) ***Use a suitable substitution to evaluate the following***;

1. $\int \frac{x}{(x^2+2)^2}\,dx$
2. $\int \frac{x-1}{(x+2)^3}\,dx$
3. $\int \sqrt{5+2x}\,dx$
4. $\int x(3x^2-4)^3\,dx$
5. $\int \frac{3}{\sqrt{(5-2x)^3}}\,dx$
6. $\int \frac{5}{\sqrt{1-6x}}\,dx$
7. $\int x^2\,\sqrt[3]{6-x^3}\,dx$
8. $\int \frac{\sqrt{1+Inx}}{x}\,dx$
9. $\int \frac{\sqrt{9-x^2}}{x}\,dx$
10. $\int \frac{x}{(x^2-4)^3}\,dx$
11. $\int \frac{1}{(3x+7)^4}\,dx$
12. $\int \sqrt[5]{(-3x+8)^6}\,dx$
13. $\int \sqrt{x\sqrt{x^3}}\,dx$
14. $\int x(2x-1)^3\,dx$
15. $\int (5x-2)^3dx$
16. $\int \frac{1}{x}\,In^2x\,dx$

17. $\int x^3\sqrt{1-x^2}\,dx$

18. $\int\left(1+\sqrt{x}\right)^{10}dx$

19. $\int \frac{18-9x}{\sqrt{3+4x-x^2}}\,dx$

20. $\int \frac{1-x}{(x+2)^5}\,dx$

## B) Show that;

21. $\int_0^2 \frac{1}{\sqrt{4x+1}}\,dx = 1$

22. $\int \frac{2x}{(4+3x^2)^2}\,dx = -\frac{1}{3(4+3x^2)} + c$

23. $\int_0^1 5x(1-x^2)^{\frac{3}{2}}\,dx = 1$

24. $\int \frac{12x}{(1-x^2)^{\frac{3}{2}}}\,dx = \frac{12}{\sqrt{1-x^2}} + c$

25. $\int_0^1 \frac{x}{\sqrt{16-7x^2}}\,dx = \frac{1}{7}$

26. $\int \frac{4}{x(1+4Inx)^2}\,dx = -\frac{1}{(1+4Inx)} + c$

27. $\int_0^1 \frac{9}{(2x+1)^2}dx = 3$

28. $\int_4^8 \frac{6x}{\sqrt{2x-7}}dx = 68$

29. $\int_0^5 x\sqrt{3x+1}\,dx = \frac{204}{5}$

30. $\int \frac{30x}{\sqrt{1-2x}}\,dx = -10(x+1)(1-2x)^{\frac{1}{2}} + C$

## C) Find the following integrals.

31. $\int \frac{1}{x^3}\sin\left(\frac{1}{x^2}\right)\,dx$

32. $\int \cot(5x-4)\,dx$

33. $\int \frac{10}{\sin^2\left(\frac{x-2}{3}\right)}\,dx$

34. $\int \frac{1}{x}\cos(Inx)\,dx$

35. $\int \frac{\sin 2x}{\cos x}\,dx$

36. $\int \cot^2(3x)\,dx$

37. $\int \frac{\sin^3(3x)}{\cos 3x}\,dx$

38. $\int \frac{\tan\sqrt{x}}{\sqrt{x}}\,dx$

39. $\int \frac{\cos x}{\sqrt[3]{\sin^2 x}}\,dx$

40. $\int \sin(3mx)\,dx$

## *D)Prove out the following;*

41. $\int_0^{\frac{\pi}{2}}(1+2cosx)^3 sinx\,dx = 10$

42. $\int_0^{\frac{\pi}{3}} sec^4x\,dx = 2\sqrt{3}$

43. $\int_0^{\frac{\pi}{3}} 15(10cosx-1)^{\frac{1}{2}}sinx\,dx = 19$

44. $\int sec^2x\{1+cot^2x\}\,dx = -cotx + tanx + C$

45. $\int \frac{sinx}{\sqrt{4cosx-1}}dx = -\frac{1}{2}\sqrt{4cosx-1} + C$

46. $\int tanxsec^4x\,dx = \frac{1}{4}sec^4x + C$

47. $\int_0^{\frac{\pi}{4}}(1-sin4x)dx = \frac{1}{4}(\pi-2)$

48. $\int_0^{\frac{\pi}{4}}\cos\left(3x+\frac{\pi}{4}\right)dx = \frac{-\sqrt{2}}{6}$

49. $\int \frac{1}{1+cosx}dx = tan\frac{x}{2} + C$

50. $\int_0^{\frac{\pi}{4}}\sin\left(2x+\frac{\pi}{4}\right)dx = \frac{\sqrt{2}}{2}$

## *E)Work out the following integrals;*

51. $\int \frac{tanx}{\sqrt{1+cos2x}}dx$

52. $\int sin\frac{3}{4}xcos\frac{1}{2}x\,dx$

53. $\int cosxsin3x\,dx$

54. $\int cos\frac{3}{2}xcosx\,dx$

55. $\int tan^2x\,dx$

56. $\int tan^3x\,dx$

57. $\int sin^3\frac{x}{4}\,dx$

58. $\int cosxsin^2 2xdx$

59. $\int sin^3\frac{x}{2}cos^3\frac{x}{2}\,dx$

60. $\int cosec^2\frac{1}{16}x\,dx$

61. $\int cot^2\frac{2}{3}x\,dx$

## *F)Evaluate the following;*

62. $\int \frac{3}{100+81x^2}\,dx$

63. $\int \frac{\sqrt{2}}{x^2+2}\ dx$

64. $\int \frac{2}{\sqrt{64-4x^2}}\ dx$

65. $\int \frac{6}{(x^2-4)^{\frac{3}{2}}}\ dx$ (**use:** $x = 2sec\theta$)

66. $\int \frac{1-x}{\sqrt{x}(x+1)} dx$ (**Use**: $\sqrt{x} = tan\theta$)

67. $\int \frac{2x^2}{\sqrt{1-x^2}} dx$

68. $\int_0^1 \frac{tan^{-1}(x)}{1+x^2} dx$

69. $\int_0^{\frac{\pi}{2}} \frac{sin2x}{\sqrt{4-sin^4 x}} dx$

70. $\int \frac{x+2}{\sqrt{1-4x^2}} dx$

***G)Show that;***

71. $\int_0^{\frac{1}{2}} \sqrt{\frac{16x}{1-x}} dx = \pi - 2$ (**Use** $\boldsymbol{x = sin^2\theta}$)

72. $\int_1^{\sqrt{e}} \frac{1}{x\sqrt{1-(Inx)^2}} dx = \frac{\pi}{6}$

73. $\int_{\frac{4}{3}}^{\frac{5}{3}} \frac{x+1}{\sqrt{9x^2-16}} dx = \frac{1}{3}(1 - In2)$

74. $\int_0^1 \frac{x^2}{(x^2+1)^3}\ dx = \frac{\pi}{3}$

75. $\int_0^{\sqrt{3}} \frac{x}{x^4+9} dx = \frac{\pi}{24}$ (**Use** $\boldsymbol{x^2 = 3tan\theta}$)

76. $\int_0^1 \frac{2\sqrt{3}}{\sqrt{4\pi^2-3\pi^2x^2}} dx = \frac{2}{3}$

77. $\int_0^1 \frac{1}{(x+1)\sqrt{x}} dx = \frac{\pi}{2}$

78. $\int_0^{\frac{3}{4}} \frac{6}{\sqrt{3-4x^2}} dx = \pi$

79. $\int_0^{\frac{1}{2}} \frac{dx}{3+4x^2} = \frac{\pi\sqrt{3}}{36}$

80. $\int_{-3}^{-2} \frac{x}{\sqrt{-x^2-6x-5}} dx = \sqrt{3} + \frac{\pi}{2} - 2$

81. $\int_{-1}^{5} \sqrt{(1+x)(5-x)}\, dx = \frac{9\pi}{2}$

82. $\int_{-1}^{1} (x+3)\sqrt{7-6x-x^2}\, dx = 8\sqrt{3}$

83. $\int_0^1 \frac{x^2-3x+1}{\sqrt{(1-x^2}}\, dx = \frac{3}{4}(\pi - 4)$

84. $\int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{5x^2-12x+4}{\sqrt{(1-x^2)}} dx = \frac{4\pi}{3}$

85. $\int_4^9 \frac{1}{(4-x)\sqrt{x}} dx = In\sqrt{3}$

86. Using substitution $x^3 = 9sin^2\theta$ or otherwise, show that:

$$\int_0^1 \frac{\sqrt{x}}{\sqrt{9-x^3}} dx = \frac{2}{3} sin^{-1}\left(\frac{1}{3}\right).$$

87. By using substitution $x = sec^2\theta$, or otherwise, evaluate; $\int_1^2 \frac{dx}{x^2(x-1)^{\frac{1}{2}}}$

**H)** Express the following in partial fractions, and hence, prove out their respective definite integrals.

88. $f(x) = \frac{8x-1}{(2x-1)^2}$, $\int_1^{1.5} f(x)dx = \frac{3}{4} + 2In2$

89. $h(x) = \frac{3x-10}{x^2+5x-6}$, $\int h(x)dx = 4In(x+6) - In(x-1) + C$

90. $p(x) = \frac{5}{3x^2-5x}$, $\int_3^5 p(x)dx = In\left(\frac{3}{2}\right)$

91. $f(x) = \frac{2x+2}{(1-x)(1-2x)}$, $\int_{1.5}^{2} f(x)dx = 7In2 - 3In3$

92. $g(x) = \frac{32-17x}{(x+1)(3x-4)^2}$, $\int_0^1 g(x)dx = 1 + 3In2$

93. $h(x) = \frac{2x^2-3}{(x-1)^2}$, $\int_2^3 h(x)dx = \frac{3}{2} + 4In2$

94. $f(x) = \frac{3x^2-2x+3}{2x(x-1)^2}$, $\int_4^9 f(x)dx = \frac{5}{24} + 2In2$

95. $m(x) = \frac{12x^2+x+3}{(6x+1)(2x^2+1)}$, $\int_0^2 m(x)dx = In\sqrt{39}$

96. $j(x) = \frac{1}{(x+x^4)}$, $\int_{\frac{1}{2}}^2 j(x)dx = In2$

97. $f(x) = \frac{70}{x(x-2)(x+5)}$, $\int_3^4 f(x)dx = 11In3 - 15In2$

98. $h(x) = \frac{x^2+3}{(x-1)}$, $\int_2^4 h(x)dx = 8 + 4In3$

99. $g(x) = \frac{x-5}{(x^2+5x+4)}$, $\int_0^2 g(x)dx = In\left(\frac{3}{8}\right)$

100. $f(x) = \frac{1}{9cos^2x-sin^2x}$, $\int_0^{\frac{\pi}{4}} f(x)dx = \frac{1}{6}In2$

101. $h(x) = \frac{(x-2)(x^2-5x-1)}{(x-1)(x-3)}$, $\int_{1.5}^2 h(x)dx = \frac{5}{4} - In6$

102. $h(x) = \frac{4x}{1-x^4}$, $\int_0^{\frac{1}{2}} h(x)dx = In\frac{5}{3}$

103. $n(x) = \frac{3x^2+1}{2x^3+x}$, $\int_1^2 n(x)dx = \frac{1}{2}In48$

104. $f(x) = \frac{x^2+3x+36}{x^3+9x^2+9x+81}$, $\int f(x)dx = In(x+9) + tan^{-1}\left(\frac{x}{3}\right) + C$

105. $f(x) = \frac{x+3}{(x+1)(x^2+4x+5)}$, $\int_0^1 f(x)dx = In\sqrt{2}$

106. $g(x) = \frac{9}{x^3+1}$, $\int_0^1 g(x)dx = 3In2 + \pi\sqrt{3}$

107. $f(x) = \frac{x+2}{(x+1)(x^2+4)}$, $\int_0^2 f(x)dx = \frac{\pi}{8} + In3$

**I)** Show that;

108. $\int_0^{In2} (e^x + 2e^{-x})^2 dx = 3 + 4In2$

109. $\int_0^{In2} 4xe^{2x} dx = -3 + 8In2$

110. $\int_0^{\frac{\pi}{2}} 4x^2 cosx dx = \pi^2 - 8$

111. $\int_0^{\frac{\pi}{4}} 4xsec^2xdx = \pi - 2In2$

112. $\int \frac{4Inx}{x^3}dx = \frac{1+2Inx}{x^2} + C$

113. $\int_1^{\frac{\pi}{3}} 2sinx\{In(secx)\}dx = 1 + In\frac{1}{2}$

***J) Using substitution $t = tan\frac{x}{2}$, prove out the following;***

114. $\int secxdx = In\{secx + tanx\} + C$

115. $\int cosecxdx = In\{cosecx - cotx\} + C$

116. $\int_0^{\frac{2\pi}{3}} \frac{1}{5+4cosx}dx = \frac{\pi}{9}$

117. $\int_0^{\frac{\pi}{2}} \frac{3\sqrt{3}}{2-cosx}dx = 2\pi$

118. $\int_0^{\frac{\pi}{2}} \frac{1}{1+sinx}dx = 1$

119. $\int_0^{\frac{\pi}{2}} \frac{1}{5+3sinx+4cosx}dx = \frac{1}{6}$

120. $\int_0^{\frac{\pi}{2}} \frac{2}{(1+sinx+2cosx)}dx = In3$

121. $\int_0^{\frac{\pi}{8}} \frac{\sqrt{3}}{2+sin4x}dx = \frac{\pi}{12}$

122. $\int_0^{\frac{\pi}{2}} \frac{1}{1+sinx+cosx}dx = In2$

123. $\int_0^{\frac{\pi}{4}} \frac{dx}{1+sin2x} = \frac{1}{2}$

(**NOTE**: *As a teacher, let a learner well-know the derivation of **101** and **102**, as it'll help him/her to become conversant with application of t – formular in change of variables*)

***K)Using substitution $t = tanx$, show that;***

124. $\int_0^{\frac{\pi}{4}} \frac{18}{3cos^2x+sin^2x} dx = \pi\sqrt{3}$

125. $\int_0^{\frac{\pi}{4}} \frac{10}{2-tanx} dx = \pi + 3In2$

126. $\int_0^{\frac{\pi}{4}} \frac{1}{cos^2x+25sin^2x} dx = \frac{1}{5} tan^{-1}(5)$

127. $\int_0^{\frac{\pi}{3}} \frac{1}{1+8cos^2x} dx = \frac{\pi}{18}$

***L) Find the following***

128. $\int_0^{\frac{\pi}{2}} \frac{x}{1+cosx} dx$

129. $\int \sqrt{x}\, tan^{-1}\sqrt{x}\, dx$

130. $\int_0^{\frac{\pi}{4}} \frac{4}{1+sin2x} dx$

131. $\int_3^9 \frac{log_{27}^{x}}{x} dx$

132. $\int \frac{In(tanx)}{sinxcosx} dx$

133. $\int_0^{\frac{\pi}{2}} sin\sqrt{x}\, dx$

# DIFFERENTIAL EQUATIONS

## a) *Formation of differential Equations*

Form differential equations for each of the following equations and in each case, state the order and degree of the DE formed.

134. $y = Ae^{-4t}$

135. $x = Ae^{3t} + Be^{-2t}$

136. $y = Acos3t + Bsin3t$

137. $x = Pe^{-2t} + Re^{2t}$

138. $y = Asin4t + Be^{-2t}$

## b) *Solving differential Equations:*

139. $x^2 \frac{dy}{dx} + xy = 3x^2 + 4y^2$

140. $\frac{dy}{dx} = \frac{4x}{4x-y}$

141. $\frac{dy}{dx} - 3 = x^{-2}y^2$

142. $2x\frac{dy}{dx} = y(x^2 - y^2)^{-\frac{1}{2}}$

143. $\frac{dy}{dx} = \sqrt{\frac{y}{x+1}}, \qquad y(3) = 9$

144. $\frac{dy}{dx} = \frac{sin^2x}{y^2}, \qquad y(\pi) = 1$

145. $\frac{dy}{dx} + ycotx = cos3x, \;\; y\left(\frac{\pi}{6}\right) = 1$

146. $xdy - 3ydx = x^3dx$

147. $4\frac{dy}{dx} + 5y = e^{\frac{1}{4}x}$

148. $cosx\frac{dy}{dx} - 2ysinx = 1$

149. Show that the general solution to the differential equation;

$$\frac{dx}{dt} = \frac{1+x^2}{1+t^2} \text{ is } x = \frac{k+t}{1-kt} \text{ where } k \text{ is a constant.}$$

150. Find y in terms $x$, given that; $x\frac{dy}{dx} = cos^2y$ and $y\left(\frac{\pi}{3}\right) = 1$

151. Find the general solution of the equation: $2x\frac{dy}{dx} - (2y+1)(x+1) = 0$

152. Solve the differential equation;

$$\frac{dy}{dx} = x - \frac{2y}{x} \quad \text{given that } y(2) = 4$$

153. Solve the differential equation; $\frac{dy}{dx} = 4x - 7$ given that $y(2) = 3$

154. Solve the differential equation: $\frac{dy}{dx} + y = e^{-x}cos\frac{x}{2}$

155. Solve the differential equation: $y^2cos^2x = tanx\frac{dy}{dx}$

156. Solve: $2\frac{dy}{dx} = e^{x-2y}$ given that y=1 when $x = 2$

157. Solve the differential equation: $(x+2)\frac{dy}{dx} = (2x^2+4x+1)(y-3)$ given that $x = 0$ when $y = 7$.

## APPLICATIONS OF DIFFERENTIAL EQUATIONS

158. In a chemical reaction, the mass, m of the product after time t minutes satisfies the differential equation $\frac{dm}{dt} = \frac{m}{t(1+t^2)}$. Two minutes after the reaction had started, the mass of the chemical produced was 10 grammes, show that: $m = \frac{5t\sqrt{5}}{\sqrt{1+t^2}}$.

159. The number, $x$ thousands, of reported cases of an infectious disease, t months after it was first reported, is now dropping. The rate at which it is dropping is proportional to the square of the number of the reported cases.

a) Form a differential equation in terms of $x$, t and a proportionality constant k.
   Initially there were 2500 reported cases and one month later, they had dropped to 1600 cases.

b) Solve the differential equation to show that $x = \frac{40}{9t+16}$ .

c) Find after how many months there will be 250 reported cases.

160. In established forest fire, the rate at which the proportion of the area X is destroyed is proportion to the product of the area destroyed and that of the remaining proportion. A particular fire is initially noticed when one half of the forest is destroyed, and it is found that the destruction rate at this time is such that if it remained constant thereafter, the forest would be completely destroyed in a further 24 hours.

i) Form a differential equation for the rate of destruction of the forest

ii) Solve the equation, and show that approximately 73% of the forest is destroyed 12 hours after its first noticed.

161. A machine depreciates at a rate proportional to its current value. Initially, the machine is valued at shs.2.5 Millions. 5 years later, it was valued at shs. 1.87 Millions. If $\theta$ is the value of the machine after t years, form a differential equation and use it to find;

i) The value of the machine after 15 years

ii) The number of years it will take the machine to be valued shs.0.5 Millions.

162. Certain bacteria in a culture are thought to increase at a rate proportional to the number present in a colony. If the number doubles in 10 years, find by what factor the initial population has been multiplied after further 20 years.

163. A body which is at a higher temperature than its surroundings cools according to the Newton's law of cooling $\theta = \theta_0 e^{-kt}$, where $\theta_0$ is the original excess of temperature and $\theta$ is the excess temperature after t minutes.

i) Show that $\frac{d\theta}{dt}$ is proportional to $\theta$

ii) If the original temperature of the body is $100^0C$, the temperature of the surrounding is $20^0C$ and the body cools to $70^0C$ in 10 minutes, find the value of k

iii) At what rate is the temperature decreasing after 20 minutes?

164. A laboratory investigation was carried out to investigate the effect of a newly produced drug on a certain poultry bacteria. It was revealed that the rate at which the

bacteria is killed is directly proportional to the population present at that time. Initially, the population was $P_0$ and at t months, it was found to be P.

i). Obtain the differential equation connecting P and t, and solve it.

ii). If bacterial population reduced to two thirds of the initial population 6 months later, solve the equation in i) above.

iii). Find how long it'll take for only 10% of the original population to remain.

165. A man starts to climb a mountain whose height is 1000m above its foot. He notices that the rate at which the temperature drops with height is directly proportional to the height. The temperature is $16^0$C at the foot and drops to $-9^0$C at the top of the mountain. Find the height at which the temperature reaches the freezing point of water.

166. A plague wipes out a community at a rate proportional to population present at any time t. If the original population is 4Millions, and the population reduces from 2.5 to $\frac{1}{5}$ Millions in 5 years. Find how long it takes to reduce the original population to 1Million.

167. A plant grows in a pot which contains volume, V of the soil. At a time, t, the mass of the plant is $m$ and the volume of the soil utilized by the roots is $\alpha m$ where $\alpha$ is a constant. The rate of increase of the mass of the plant is proportional to the mass of the plant times the volume of the soil not yet utilized by the roots.

a) Obtain a differential equation and verify that it can be written in the form;

$V\beta \frac{dt}{dm} = \frac{1}{m} + \frac{\alpha}{V - \alpha m}$ where $\beta$ is also a constant.

b) Find in terms of V and $\beta$, the time taken for the plant to double its mass, if initially, mass of the plant is $\frac{V}{4\alpha}$.

168. The rate at which a candidate was losing support during an election campaign was directly proportional to the number of supporters he had at that time. Initially, he had $V_0$ supporters and t-weeks later, He had V supporters.

i) Form a differential Equation connecting V and t

ii). Given that the supporters reduced to two thirds of the initial number in 6 weeks, solve the equation in i) above.

iii). Find how long it'll take for the candidate to remain with 20% of the initial supporters?

169. An Epidermic is spreading through a community at a rate which is proportional to the product of number of people who have contracted it and those who have not yet contracted it. Given that $x$ is the portion of the people who have contracted the epidemic at time, t,

a) Form a differential equation relating $x, t$ and a constant, k
b) Show that, if initially, a portion $P_0$ of the population had contracted the epidermic, then;

$$x = \frac{P_0}{P_0 + (1 - P_0)e^{-kt}}.$$

170. According to Newton's law of cooling, the rate at which a hot object cools is proportional to the difference between the temperature of the body and that of the

surrounding air (assumed constant). If an object cools from $100^0$C to $80^0$C in 10 minutes and from $80^0$C to $65^0$C in another 10 minutes, find the temperature of the;

a) Surrounding.

b) Object after a further 10 Minutes.

171. A liquid cools in an environment of a constant temperature of $21^0$C at a rate proportional to the excess temperature. Initially, the temperature of the liquid is $100^0$C and after 10 minutes, the temperature drops by $16^0$C. Find how long it takes for the temperature of the liquid to be $70^0$C

172. A moth ball evaporates at a rate proportional to its volume, losing a half of its volume every 4 weeks. If the volume of the moth ball is initially $15cm^3$ and becomes ineffective when its volume reaches $1cm^3$, how long is the moth ball effective?

173. A container in the shape of an inverted right circular cone, the height of the cone is 12cm and its vertical angle is $60^0$. A tap delivers water into the container at a rate which is directly proportional to the depth of water collected at any time t.

a) Show that the rate at which the water level is rising is inversely proportional to the depth of the water collected.

b) If it takes one (1) minute to collect a depth of 6cm of water, calculate the time in minutes it takes to fill the whole container.

174. In a certain type of a chemical reaction, a substance A is continuously transformed into a substance B. Throughout the reaction, the sum of masses of A and B remains constant and equal to $m$. The mass of B present at time, t after the commencement

of the reaction is denoted by $x$. At any instant, the rate of increase of the mass of B is $k$ times the mass of A where $k$ is a positive constant.

a) Form a differential equation relating $x$ and t. Hence, solve the equation, given that $x = 0$ when $t = 0$.

b) Given also that $x = \frac{1}{2}m$ when $t = In2$, determine the value of $k$, and show that at time t, $x = m(1 - e^{-t})$.

c) Hence, find the value of;

i) $x$ (in terms of $m$) when $t = 3In2$.

ii) $t$ when $x = \frac{3}{4}m$.

175. In Mitooma town, the rate at which buildings are collapsing is proportional to those that have already collapsed. If initially the number of buildings that had already collapsed is $N_0$,

a) Show that $N = N_0 e^{kt}$ where $k$ is a constant.

b) If the number of collapsed buildings doubled the initial number in 10 years, find the value of $k$

c) By what factor of $N_0$ will the buildings have collapsed after 30 years?

176. The mass $m$ grammes of a burning candle, t-hours after it was lit, satisfies the differential equation: $\frac{dm}{dt} = -k(m - 10)$ where $k$ is a positive constant. If the total mass of the candle was initially 120 grammes, and three hours later, its mass had halved, show that;

a) $k = \frac{1}{3} In\left(\frac{11}{5}\right)$

b) Mass of the candle after further period of 3 hours had elapsed is approximately 32.7 grammes.

177. A cylindrical tank of height 150cm is full of oil which started leaking from a small hole at the side of the tank. If the leaking of the tank is defined by a differential equation:

$$\frac{dh}{dt} = \frac{1}{4}(h-6)^{\frac{3}{2}}$$

a) Show that: $t = \frac{8}{\sqrt{h-6}} - \frac{2}{3}$

b) State how high is the hole from the bottom of the tank. Hence, show further that it takes 200 seconds for the oil level to reach 4cm above the hole level. (**Ans:** 6cm)

178. The temperature in a bathroom is maintained at the constant value of $20^0$C and the water in a hot bath is left to cool down. The rate at which the temperature of the water in the bath is cooling down is proportional to excess temperature.

Initially the bathwater had a temperature of $40^0$C, and at that instant, it was cooling down a rate of $0.005^0$C per second.

a) Show that: $\frac{dT}{dt} = -\frac{1}{4,000}(T-20)$

b) After how long the temperature of the bathwater will drop to $36^0$C?

179. At time t hours, the rate of decay of the mass, $x$ kg, of a radioactive substance is directly proportional to the mass present at that time. Initially the mass of the radioactive substance is $x_0$.

a) Show that: $x = x_0e^{-kt}$ where $k$ is a positive constant.

b) Find the value of $t$ when $x = \frac{1}{2}x_0$

180. Mould is spreading on a wall of area 20m$^2$. When it was first noticed 2m$^2$ of the wall was already covered by this mould. Let A, in m$^2$ represent the area of the wall covered by the mould, after time t weeks.

The rate at which A is changing is proportional to the product of the area covered by the mould and the area of the wall not yet covered by the mould. After a further period of 2 weeks the area of the wall covered by the mould is 4m$^2$. Show that:

$$A = \frac{20}{1+9\left(\frac{2}{3}\right)^t}.$$

181. The mass of a radioactive isotope decays at a rate proportional to the mass of the isotope present. The half-life of the isotope is 80 years. Determine the percentage of the original amount which remains after 50 years. (**Ans:** 64.8%)

182. A small forest with an area of 25km$^2$ has caught fire and it burns at a rate proportional to the difference between the total area of the forest squared, and the area of the forest destroyed squared. When the fire was first noticed 7km$^2$ of the forest had been destroyed and at that instant the rate at which the area of the forest was destroyed was 7.2km$^2$ per hour. Show clearly that;

a) $\frac{dA}{dt} = \frac{1}{80}(625 - A^2)$, where A is the area of the forest destroyed by the fire, t hours after the fire was first noticed.

b) 14km$^2$ of the forest will be destroyed, approximately 66 minutes after the fire was first noticed.

183. A variable $x$ decreases with time t, both in suitable units, at a rate directly proportional to the value of $x^3$ at that time. If the value of $x$ is half of its initial value

when t = 3, determine the value of t when $x$ has reduced to 20% of its initial value. (**Ans**: t=24)

184. The number $x$ of bacterial cells in time t hours, after they were placed on a laboratory dish, is increasing at the rate proportional to the number of the bacterial cells present at that time.

a) If $x_0$ is the initial number of the bacterial cells and $k$ is a positive constant, show that:

$$x = x_0 e^{kt}$$

b) If the number of bacteria triples in 2 hours, show that: $k = In\sqrt{3}$.

## MISCELENEOUS QUESTIONS

185. $\int \frac{sec^2 t}{1+tant} dt$

186. $\int \frac{tan\theta}{cos^2\theta} d\theta$

187. $\int tan^2 x secx dx$

188. $\int \frac{x^{\frac{1}{3}}}{\left(x^{\frac{1}{2}}+x^{\frac{1}{4}}\right)} dx$

189. $\int \frac{cosx}{\sqrt{4-sin^2 x}} dx$

190. $\int \frac{tanx}{In(cosx)} dx$

191. $\int cosx\sqrt{4-sin^2 x}\, dx$

192. $\int \frac{x}{\sqrt{4x-x^2}} dx$

193. $\int \frac{x^7}{\sqrt{1-x^4}} dx$

194. $\int \frac{dx}{x\sqrt{6x-x^2}}$

195. $\int \frac{e^{2x}}{e^{2x}-1} dx$

196. $\int \sqrt{x^2-9}\, dx$

197. $\int sec^{-1}\sqrt{x}\ dx$

198. $\int \frac{(sin^{-1}x)^2}{\sqrt{1-x^2}} dx$

199. $\int \frac{sin^{-1}x}{x^2}\ dx$

200. $\int \frac{sin^3\theta}{cos\theta-1} d\theta$

**jkbegumisa2025@gmail.com (+256786108453/+256700798389)**

# END